$$y_t = \beta(1+X_t) + \varepsilon_t$$

$$y_t \sim N(\beta(1+X_t), \sigma^2)$$

$$\frac{\partial \ln L}{\partial a} ; \frac{\partial \ln L}{\partial b} ; \frac{\partial \ln L}{\partial \sigma^2}$$

$$L = (2\pi)^{-n/2}(\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum(y_t - \beta - \beta X_t)^2\right)$$

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum(y_t - \beta - \beta X_t)^2$$

$$\frac{\partial \ln L}{\partial \beta} = + \frac{2}{2\sigma^2}\sum(1+X_t)(y_t - \beta(1+X_t)) = 0$$

$$\frac{1}{\sigma^2}\sum(1+X_t)(y_t - \beta(1+X_t)) = 0$$

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2}\cdot\frac{1}{\sigma^2} + \frac{1}{2\sigma^4}\sum(y_t - \beta(1+X_t))^2 = 0$$

1) $\sum(1+X_t)(y_t - \beta(1+X_t)) = 0$

2) $\frac{1}{2\sigma^4}\sum(y_t - \beta(1+X_t))^2 = \frac{n}{2\sigma^2}$

$$\sum(y_t - \beta(1+X_t))^2 = n\sigma^2$$

$$\Downarrow$$

$$\boxed{\sigma^2 = \frac{1}{n}\sum e_t^2}$$

1) $\Rightarrow$ $\sum y_t(1+X_t) - \beta\sum(1+X_t)^2 = 0$

$$\boxed{\beta = \frac{\sum y_t(1+X_t)}{(1+X_t)^2};}$$